1 Beginning — Surface Area of a Sphere

The surface area of a sphere is given by the formula

$$A = 4\pi r^2,$$

where r is the radius. Write a program that takes a radius as input and produces the surface area as output. Approximate π with 3.1416. You may assume the input is a positive value.

Note: If your results are slightly different from those below, it is probably due to round-off error, which we will ignore. Your results should agree to at least 3 significant digits.

Example 1:

```
Enter radius: 3.4
The surface area is 145.267584
```

Example 2:

```
Enter radius: 1.5
The surface area is 28.2744
```

2 Beginning — Isosceles Triangles

Let $(x_1, y_1), (x_2, y_2)$, and (x_3, y_3) be three points in the plane. These points are *colinear* if $(x_1 - x_2)(y_2 - y_3) = (y_1 - y_2)(x_2 - x_3)$. They form the vertices of a triangle if they are not colinear. This triangle is *isosceles* if at least two of its edges have the same length. The length of the edge from (x_1, y_1) to (x_2, y_2) is given by

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

Write a program that takes six integers defining three points and reports whether these three points form an isosceles triangle. Note that your program does not need to compute a square root — you can use the squares of the lengths instead. Thus, it is possible (and recommended) to solve this problem without using floating-point numbers.

Example 1:

```
Enter x1: 1
Enter y1: 2
Enter x2: 3
Enter y2: 4
Enter x3: 5
Enter y3: 6
This is not an isosceles triangle.
```

Example 2:

```
Enter x1: 0
Enter y1: 0
Enter x2: 1
Enter y2: 4
Enter x3: 2
Enter y3: 0
This is an isosceles triangle.
```

Example 3:

```
Enter x1: 2
Enter y1: -1
Enter x2: 6
Enter y2: 6
Enter x3: 3
Enter y3: 2
This is not an isosceles triangle.
```

3 Beginning – Crossword puzzle

Write a program that accepts two words each less than 10 letters long. Find a common letter in the two words. Write the first word vertically and the second horizontally so the two words share their common letter. If the two words do not have a common letter, print out the message "no common letter."

Example1:

input: example school

output:

```
    e
    x
    a
    m
    p
school
    e
```

Example 2:

input: peanut school

output: no common letter

4 Beginning – Cryptography

Cryptography is about protecting information. Early schemes involved switching letters. A common approach was using an encoding wheel the "moved" every letter the same number of letters down in the alphabet. The letters on the wheel "wrap around" (a follows z).
Write a program that accepts an integer n and a word less than 10 letters long. Print out the encoded word with every letter replaced by the nth successor.

Example1:

input: 1 example
output: fybnqmf

Example 2:

input: 25 peanut
output: odzmts

5 Beginning — Collatz Sequences

Write a program that takes as input a positive integer n and prints the Collatz sequence from n, obtained as follows:

- The first element of the sequence is n.
- If the last element generated is even, the next element is obtained by dividing the last element by 2.
- If the last element generated is odd and greater than 1, the next element is obtained by multiplying the last element by 3, then adding 1.
- The sequence ends when 1 is generated.

You may assume that the input is a positive integer.

Example 1:

```
Enter n: 5
5
16
8
4
2
1
```

Example 2:

```
Enter n: 6
6
3
10
5
16
8
4
2
1
```

6 Beginning – Number Bases

Numbers can be represented in different bases. Normally, we represent numbers in a base 10 notation where the lowest digit represents the number of units, the next digit represents the number of 10's, etc. Base 2 notation (binary) is very common in computer science. The number 111 in binary represents (left to right) one 2^2, one 2^1 and one 2^0 or 4+2+1 = 7.

Write a program that accepts a number in base 10 notation and prints out the base 2 representation.

Example1:

input: 10
output: 1010

Example 2:

input: 4
output: 100

1 Advanced – Crossword puzzle

Write a program that accepts six words (or letter combinations) each 3 letters long. Determine if the six words can appear in a 3 x 3 crossword with three of the words horizontally and three vertically, and every vertical and horizontal pair sharing one letter. If this is not possible, print out the message “not possible”.

Example1:

input: tan are nan tom men ora

output:

tom

are

nan

Example 2:

input: tan are soo and hen oar

output: not possible

2 Advanced — Prime Factors

A positive integer is said to be *prime* if it has no factors other than itself and 1. Every integer larger than 1 can be factored into the product of prime numbers. Write a program that takes a integer larger than 1 as input and produces a list of its prime factors. Each factor should occur the same number of times in the list as it occurs in the factorization; in other words, the product of the values produced should equal the input value.

Example 1:

```
Enter n: 72
2
2
2
3
3
```

Example 2:

```
Enter n: 8200
2
2
2
5
5
41
```

Example 3:

```
Enter n: 13
13
```

3 Advanced — Vector Multiplication

Given an m-element column vector a and an n-element row vector b, the product ab is the $m \times n$ matrix c such that

$$c_{ij} = a_i b_j.$$

Write a program that takes positive integer values m and n, follwed by the m integer values of a and the n integer values of b, and prints the product ab. Your matrix must be formatted so that each column is right-justified with at least one blank space between columns. You may assume that no value in the matrix requires more than 11 characters and that $1 \leq m \leq 20$, $1 \leq n \leq 6$.

Example 1:

```
Enter dimension of a: 5
Enter dimension of b: 4
Enter a[0]: 1
Enter a[1]: 2
Enter a[2]: 3
Enter a[3]: 4
Enter a[4]: 5
Enter b[0]: 4
Enter b[1]: 3
Enter b[2]: 2
Enter b[3]: 1
           4           3           2           1
           8           6           4           2
          12           9           6           3
          16          12           8           4
          20          15          10           5
```

Example 2:

```
Enter dimension of a: 2
Enter dimension of b: 5
Enter a[0]: -3
Enter a[1]: 4
Enter b[0]: 5
Enter b[1]: -6
Enter b[2]: 0
Enter b[3]: 7
Enter b[4]: 2
         -15          18           0         -21          -6
          20         -24           0          28           8
```

4 Advanced – Number Systems

Numbers can be represented in different bases. Normally, we represent numbers in a base 10 notation where the lowest digit represents the number of units, the next digit represents the number of 10's, etc. Base 2 notation (binary) is very common in computer science. The number 111 in binary represents (left to right) one 2^2, one 2^1 and one 2^0 or 4+2+1 = 7.

Write a program that accepts two integers. The first number will be a base (< 10) and the second number will be interpreted as a number in that base. Print out the sum of the value of the digits in that base and then print the original number in base 10 notation.

Example1:

input: 2 101

output:

10

5

Example 2:

input: 5 4444

output:

31

624

5 Advanced – Cryptography

Cryptography is about protecting information. Some schemes involved switching the positions of letters. A common approach was to use an array and write the words consecutively along the rows and then read out the encrypted version by columns using a different order of the columns.

For example, "123 this is a test." would map into a three column array

thi
sis
ate
st

Reading down the columns, leftmost first would give "tsashittise"

Write a program that accepts an integer n that represents the sequences of the columns and a phrase less than 80 letters long. The integer 321 would specify that the array had three columns, the third column would be read first, the middle second and the leftmost column last. The message will terminate in a period which is not included in the encrypted message. Print out the encoded message without any blanks.

Note that during the contest, it was discovered that the above description is ambiguous about how the interpret the integer that represents the sequences of columns. The following examples are also ambiguous. The interpretation used in the solution is that each digit represents the ordering for that column. E.g. 312 would indicate that the second column would be first, the last column would be second and the first column would be third. An alternative interpretation would be that the third column would be first, the first column would be next and the second column would be last.

Example1:

input: 321 this is a test.
output: isehitttsas

Example 2:

input: 2134 this is a test.
output: hsstieiatst

6 Advanced — Longest Increasing Subsequence

Write a program that takes as input a positive integer n, followed by an n-integer sequence, then prints a longest increasing subsequence of the given sequence. A subsequence contains elements of the original sequence in the same order, but may skip elements. If there are more than one longest increasing subsequence, your program may print any one of them. (Thus, in Example 1 below, an alternative longest increasing subsequence would be $-4, -2, 0, 3, 7$.) You may assume that $1 \leq n \leq 20$.

Example 1:

```
Enter length of sequence: 10
Enter number 0: 5
Enter number 1: -4
Enter number 2: 7
Enter number 3: -2
Enter number 4: 6
Enter number 5: 8
Enter number 6: 0
Enter number 7: 3
Enter number 8: 9
Enter number 9: 7
The longest increasing subsequence is: -4 -2 6 8 9
```

Example 2:

```
Enter length of sequence: 5
Enter number 0: 1
Enter number 1: 2
Enter number 2: 3
Enter number 3: 4
Enter number 4: 0
The longest increasing subsequence is: 1 2 3 4
```